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SHOCK-WAVE FLOW REGIMES AT ENTRY INTO THE DIFFUSER OF A HYPERSONIC RAMJET ENGINE: INFLUENCE OF PHYSICAL PROPERTIES OF THE GAS MEDIUM

G. A. Tarnavskii

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The physical aspects of the effective-adiabatic-exponent model making it possible to decompose the total problem on modeling of high-velocity gas flows into individual subproblems ("physicochemical processes" and "aeromechanics"), which ensures the creation of a universal and efficient computer complex divided into a number of independent units, have been analyzed. Shock-wave structures appearing at entry into the duct of a hypersonic aircraft have been investigated based on this methodology, and the influence of the physical properties of the gas medium in a wide range of variations of the effective adiabatic exponent has been studied.

Introduction. The creation of new-generation high-speed aircraft and the beginning of their flights in the upper troposphere going into near-earth orbits with altitudes of about 100 km call for a different approach to the computer modeling of problems of aerodynamic. Whereas supersonic gas flow were studied earlier under the assumption (model) that the physical properties of the gas medium are constant in all the flow zones under study, the extension of modeling to the region of hypersonic velocities requires that this model be revised and the actual properties of the gas, in particular the Earth's atmosphere, with their change due to the occurrence of different physicochemical processes be allowed for.

Such a "complete" formulation of the problem leads to extremely high requirements imposed on computers, which exceed their modern capabilities. Furthermore, the creation of universal algorithms and computer programs requires that the total problem be decomposed into a number of subproblems: "aerodynamics," "physical processes," and "chemical reactions" (see [1] for details). In addition to the efficiency of realization of such computational algorithms on single-processor computers, the structurization of the problem makes it possible to carry out its "paralleling" and to ensure the application of multiprocessor systems to the solution of various problems of gas dynamics (see [2]).

Physical Model. In the present work, we use the model of "effective adiabatic exponent" [5–7] to allow for the physical processes in the gas medium (see [3, 4]). This method has successfully been used for modeling of a wide range of problems of aerodynamics [8–11]. We briefly present and analyze its essence. The first basic proposition of the model is that the "physics" of the problem is separated from "aeromechanics" in an algorithmical sense. The second basic proposition is that the relation between these segments (subproblems) in a program system is carried out in terms of a special quantity — an effective adiabatic exponent γ_{eff} which is a function of the pressure p ad the temperature T, i.e., a quantity that varies throughout the flow region (in a program sense, a 2D or 3D array depending on the dimension of the problem). It is precisely this that differentiates γ_{eff} from the "ordinary" adiabatic exponent γ, i.e., the quantity constant throughout the flow region (scalar in a program sense) and invariant with time. The third basic proportion is that γ_{eff} is computed, in one manner or another, in accordance with the physics of the process (see [7] with allowance for [3–5]) or is taken from certain tables (e.g., [12]) or electronic databases. Computation of γ_{eff} may be carried out by the thermodynamic methods [5, 13]

$$\gamma_{\text{eff}}(p, T) = -\left(\frac{\partial \ln p}{\partial \ln V}\right)_S = -\frac{V}{p}\left(\frac{\partial p}{\partial V}\right)_S \tag{1}$$

Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences, 6 Akad. Lavrent'ev Ave., Novosibirsk, 630090, Russia; email:Gennady@Tarnavsky.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 4, pp. 69–80, July–August, 2006. Original article submitted January 11, 2005.

for a certain, not necessarily ideal, equation of state $p = p(V, T)$. In particular, γ_{eff} = const (in this case $\gamma_{eff} = \gamma$ by definition) follows from (1) for the equation of state of an ideal and perfect gas $p = RT/V$ after simple but cumbersome transformations by the Jacobian technique with the use of the thermodynamic identity $\partial(p, V)/\partial(T, S) = 1$.

In the context of an analysis of the physicochemical processes in a gas medium, it is more convenient to compute γ_{eff} by the methods of statistical physics [13, 14] in terms of the classical adiabatic exponents γ_i of the gas components (considered as being mutually independent) constituting the gas mixture and their concentrations α_i in the gas medium:

$$\gamma_{eff}(p, T) = \sum_i \alpha_i \gamma_i , \quad \sum_i \alpha_i = 1 . \tag{2}$$

We emphasize that by the components of the gas medium in (2) we mean not only the chemical constants but also the physical constants — atoms and molecules in different states. The adiabatic exponent of the ith component of the gas medium with excitation level F_i is determined [6, 7] in terms of the number of excited degrees of freedom f_{in} of this component (with allowance for the statistical weight g_{in} of this degree of freedom):

$$\gamma_i = 1 + 2/F_i , \quad F_i = \sum_n g_{in} f_{in} . \tag{3}$$

We give statistical weights for different degrees of freedom: $g = 1$ for translational and rotational degrees of freedom and $g = 2$ for vibrational ones (since energy in them can exist in two forms: as potential or kinetic energy). In the excitation of electron shells, the statistical weight of the state is equal to the number of electrons on the outer shells of this component of the medium. In particular, for inert gas (argon and xenon) used as working media in aerodynamic experiments we have $g = 8$. Let us compute $\gamma_i(p, T)$ for certain gases.

Monatomic Gases. Only three (by the number of possible coordinate directions of motion) translational degrees of freedom are excited at low temperatures; consequently, according to (3), we have $n = 3$, which yields $F_i = 3$ and $\gamma_i = 5/3$. Since all the three translational degrees of freedom of any component of any gas medium are invariably excited (the case of the absolute thermodynamic zero of temperature is not considered), from (2), with allowance for the homogeneity of the medium $i = 1$, we have $\gamma_{eff} = 5/3 \approx 1.67$ for the entire gas. This value is well known in classical aerodynamics. At high temperature, when the electron degrees of freedom are excited in addition to the translational ones, from (3) we have $n = 4$, $g_{i1} = g_{i2} = g_{i3} = 1$, $g_{i4} = 8$, and $f_{i1} = f_{i2} = f_{i3} = f_{i4} = 1$, from which $F_i = 11$ and $\gamma_i = 11/9 \approx 1.18$. The value of the effective adiabatic exponent γ_{eff} as a function of the temperature and the pressure p of the gas medium will invariably lie in the interval 1.67–1.18. This temperature determines the level of excitation of the electron shells of the gas $\alpha(p, T)$, and, according to (2), the adiabatic exponent of the medium will be obtained from the formula

$$\gamma_{eff} = 1.18\alpha + 1.67\,(1 - \alpha) . \tag{4}$$

The value of $\alpha(p, T)$ may be determined from formulas of the Arrhenius or Saha type

$$\frac{\alpha^2}{1 - \alpha} = A \exp\,(-\,W/kT) , \tag{5}$$

where W is the potential of excitation of the electron degrees of freedom and A is the preexponential factor, namely, the constant in simple models or the density and/or temperature function in more complex models (see [3–7, 13, 14] for greater detail).

We note that in experimental works [15, 16] in which high-temperature hypersonic flows have been investigated, values of $\gamma = 1.22$ in [15] and $\gamma = 1.25$ in [16] were determined (on certain zones of flow, the temperature exceeded 10,000 K).

Diatomic Gases. In connection with the important circumstance that diatomic gases — the oxygen O_2 and the nitrogen N_2 — form the basis for the Earth's atmosphere (20 and 78%), we should analyze the dependence $\gamma_{eff}(p, T)$

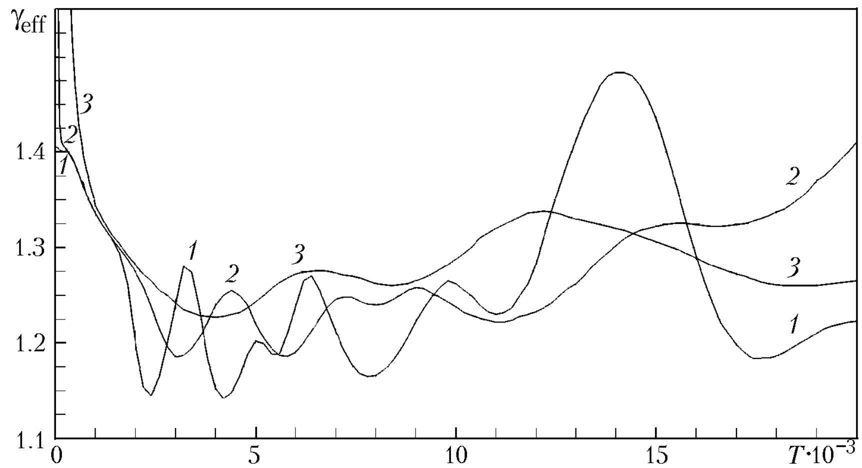

Fig. 1. Ratio $\gamma_{eff} = c_p/c_v$ (for air) vs. temperature in variation of the pressure: 1) 0.001; 2) 1; 3) 1000 atm.

in detail. Only three translational degrees of freedom are excited, when T values are very low (near the absolute zero), and $\gamma_{eff} = 5/3$, just as for a monatomic gas. The rotational degrees of freedom are excited for O_2 and N_2, when T values are low (5 to 50 K depending on the specific value of p). Since O_2 and N_2 are linear molecules, there are only two possible independent directions of the axes of rotation (two degrees of freedom). Then (3) yields $F_i = 5$ and $\gamma_i = 7/5 = 1.4$ for both O_2 and N_2. Next it follows from (2) that we have $\gamma_{eff} = \Sigma(\alpha_i \cdot 1.4) = 1.4\Sigma\, a_i = 1.4$ for O_2 and N_2 both taken separately and in a gas mixture (air) irrespective of the contractions of O_2 and N_2. This is a γ value well known in classical aerodynamics.

Let us consider the change in γ_{eff} with increase in T, for example, for O_2. A vibrational degree of freedom is excited in an oxygen molecule, as T grows. Such the molecule is linear, and there is only one direction of vibrations (along the straight line connecting atomic centers), i.e., only one degree of freedom (we recall that is statistical weight is equal to 2; the terminology "vibrational degrees of freedom are half-excited" is sometimes used). Expressions (3) yield a chain of values (three translational, two rotational, and one vibrational degree of freedom): $F_i = 1 \cdot 3 + 1 \cdot 2 + 2 \cdot 1 = 7$ and $\gamma_{eff} = 9/7 \approx 1.28$. The effective adiabatic exponent of the gas in (2) is determined by the level of excitation of vibrations of the gas molecules $\alpha_1(p, T)$. This value in turn is found from (5), where by W is meant the potential of excitation of vibrations in O_2. Next, we have the dissociation of O_2 molecules into O atoms ($\gamma_{eff} = 5/3$ for a monatomic gas) with increase in T (in the range 2000–4000 K, as a function of p). The degree of a totally dissociated gas $\alpha_2(p, T)$ may also be determined from a formula of the type (5), where by W is meant the dissociation potential. Thus, the process of excitation of vibrations in O_2 decreases, whereas the process of dissociation (disintegration of molecules $O_2 \to O + O$) increases the value of γ_{eff}:

$$\gamma_{eff} = 1.28\alpha_1 + 1.67\alpha_2 + 1.4\,(1 - \alpha_1 - \alpha_2)\,. \tag{6}$$

Next, as T increases, the process of excitation of the electron shells (first of the outer shell) of O atoms and the remaining O_2 molecules followed by the process of ionization proper (separation of electrons from the shell and the formation of ions $O \to O^+ + e$) begins. The dependence $\gamma_{eff}(p, T)$, on the whole, has a wave character with minima (for the maximum excitation of vibrations in O_2 molecules and for the maximum excitation of electron shells in O atoms) and maxima (total dissociation of O_2 into O and single, double, etc. ionization to form O^+ an O^{++}).

An analogous change in $\gamma_{eff}(p, T)$ occurs for the nitrogen N_2 but with other values of $\alpha_1(p, T)$ and $\alpha_2(p, T)$, which leads to a shift of the minima and maxima of $\gamma_{eff}(p, T)$ for N_2 compared to O_2. The course of the chemical reactions to form new di- and triatomic compounds of the type of NO, NO_2, etc. makes the pattern of the dependence $\gamma_{eff}(p, T)$ more complex.

The dependence $\gamma_{eff}(p, T)$ for air is presented in Fig. 1 in parametric form (the parameter is the pressure p). The data have been taken from [6, 7] with allowance for the tables of [12].

Calculation of $\gamma_{eff}(p, T)$ is a very difficult problem taking much computer time. This precisely is the reason, as has already been discussed, why the optimum method of modeling hypersonic flows is the sedimentation of the total problem into two subproblems: "physics" in which the physicochemical processes are modeled, and "aeromechanics," in which one can model various gasdynamic problems with their interface, in particular, through the value of the effective adiabatic exponent $\gamma_{eff}(p, T)$, transferred from the segment "physics" to the segment "aeromechanics." The values of the pressure p and the temperature T are transferred in the opposite direction.

Problem under Study. The effective-adiabatic-exponent model considered above was applied to approximate allowance for the physicochemical processes exerting a substantial influence on the gasdynamic characteristics of the elements of new-generation aircraft under development.

The present work primarily seeks to study the interaction of shock waves, for example, in the air intakes and nozzles of hypersonic-aircraft engines, in wide ranges of flight regimes. For the hypersonic ramjet engine to operate in the nominal regime it is necessary to create a system of correction of the entry of the flow into the diffuser. Such systems are mechanical, as a rule, and provide a possibility of varying entrance angles. The idea of "thermal correction" of the diffuser [17], which assumes the supply of energy to the freestream in front of the diffuser, seems very promising. However, it should be noted that such a correction (just as mechanical one) cannot assure the absence of off-nominal regimes in all cases, much less in maneuvering of an aircraft. One off-nominal regime is that of incidence of an oblique (skew) shock into the diffuser and of its reflection, which may cause the separation of the flow and the formation of stagnation or recirculation zones of flow and its substantial inhomogeneity and may lead to high thermal and force loads. Therefore, it is of prime importance to study such regimes for different altitudes of flight and flying speeds and to predict the consequences of their occurrence.

For high-speed aircraft, the supply of an oxidizer (air) to the duct of a hypersonic ramjet engine with its precompression is entirely, in fact, determined by the flying speed and the diffuser geometry which must ensure the stability and predictability of functioning in addition to the optimality of air intake. A system of oblique shocks determining the structure of the gas flow in the duct is realized at entry into the diffuser of the hypersonic ramjet engine. The development of methods of mathematical modeling due to modern computers has made it possible to study spatial high-enthalpy gas flows to form complex shock-wave structures in the flow, including the case where we have dualism of solution — the possibility of shock-wave patterns of reflection of two types: regular or Mach reflection (Neumann paradox) existing for the same governing parameters. It becomes quite important to study the problems of nonuniqueness and hysteresis of the resulting numerical solutions and to analyze their adequacy to actual physical processes.

Investigations of the regular and Mach reflection of shock waves carried out at present enable us to draw certain conclusions on the domains of their existence, including the domains of existence of a dual solution, i.e., the presence of a number of subranges of variation in the governing parameters of the process, such as the Mach number of the freestream, the angle of deflection of the flow, etc., for which stable patterns of both regular reflection and Mach reflection may be formed. These two shock-wave structures occurring in reflection of a shock wave in steady-state flows are shown diagrammatically in Fig. 2.

The regular-reflection pattern (Fig. 2a) formed in incidence of a supersonic flow with Mach number M_0 on two wedges with angles β_1 and β_2 includes respectively two oblique shocks i_1 and i_2 formed near the wedge surface and falling into the flow region with angles of inclination φ_1 and φ_2 (here and in what follows the angles are determined in relation to the direction of the freestream vector) and two reflected shocks r_1 and r_2 with angles of inclination φ_3 and φ_4. These shocks intersect at point R. The wake S with an angle of inclination δ is formed in transmission of the flow by a system of shocks with angles of deflection of the flow θ_1, θ_2, θ_3, and θ_4 on the shocks i_1, i_2, r_1, and r_2 respectively. The relations $\theta_1 = \beta_1$, $\theta_2 = \beta_2$, and $\theta_1 - \theta_3 = \theta_2 - \theta_4 = \delta$ hold true for the stationary pattern. For symmetric ($\beta_1 = \beta_2$) reflection, we have $\delta = 0$.

When a wave structure with Mach reflection occurs (Fig. 2b), a central shock m whose curvilinear front links two triple points of intersection of the shocks (i_1, r_1, m) and (i_2, r_2, m) appears in addition to the incident and reflected shocks, and two wakes S_1 and S_2 with angles of inclination δ_1 and δ_2 occur, too. The relations $\theta_1 = \beta_1$, $\theta_2 = \beta_2$, $\theta_1 - \theta_3 = \delta_1$, and $\theta_2 - \theta_4 = \delta_2$ hold true for the stationary pattern. In the case of symmetry ($\beta_1 = \beta_2$) it is clear that $\theta_1 - \theta_2$ and $\delta_1 = \delta_2 = 0$.

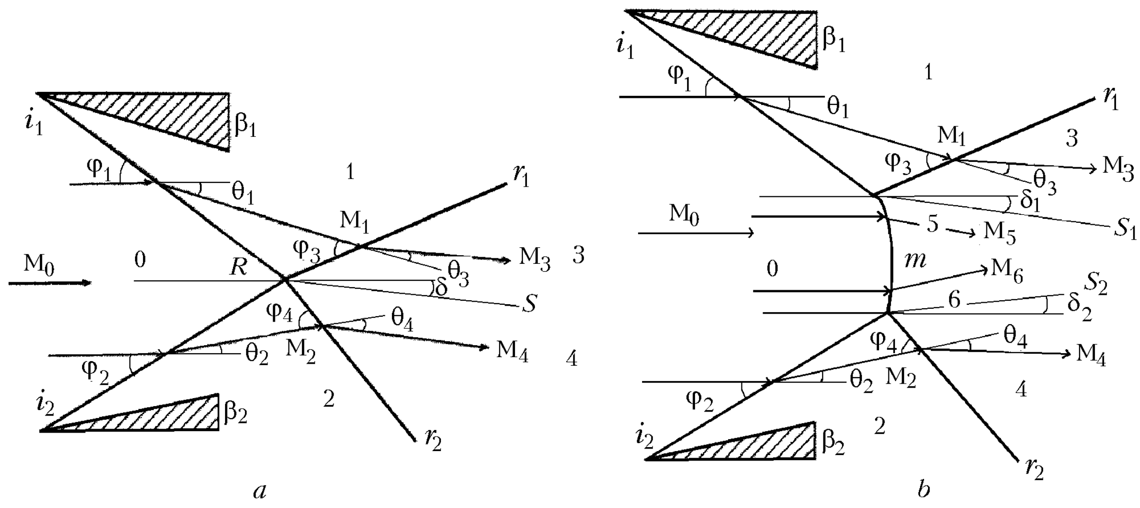

Fig. 2. Patterns of shock-wave structure in interaction of shocks: regular (a) and Mach (b) reflections.

The entire region of flow is subdivided into a number of zones (see Fig. 2), with the flow (homogeneous one, in an idealized formulation) having its own characteristics in each zone. Zone 0, the region of undisturbed flow, is bounded by any line placed in the region of a supersonic freestream (e.g., by the straight line linking the tops of the wedges) on the left and by the fronts of the shocks i_1 and i_2 (and additionally by the front of the shock m for Mach reflection) on the right. Zone 1, the region of flow turned (clockwise) to the shock i_1 along the surface of the upper wedge, is bounded by the fronts of the shocks i_1 and r_1 on the left and on the right respectively. Analogously zone 2, the region of flow turned (counterclockwise) to the shock i_2 along the surface of the lower wedge, is bounded by the fronts of the shocks i_2 and r_2 on the left and on the right respectively. Zone 3, the sector of flow turned (counterclockwise) to the shock r_1, is bounded by its front and the surface of contact discontinuity which is boundary of the wake S (S_1 for Mach reflection). Zone 4, the sector of flow turned (clockwise) to the shock r_2, is bounded by its front and the surface of contact discontinuity which is also a boundary of the wake S (S_2 for Mach reflection). In the case of regular reflection zones 3 and 4 shake the boundary (directly join together), whereas in the case of Mach reflection there are zones 5 and 6 between them, which are the regions of flow behind the front of the shock m.

The transitions between these two types of reflection are determined by he separation criterion and the Neumann criterion. These criteria (bifurcation points) differentiate between three regions in which the existence just of Mach reflection (in the first region), of both Mach reflection and regular reflection (in the second one), and just of regular reflection (in the third region) is possible. The process of transition of these types of reflection with variation of the parameters governing the physics of the problem, e.g., the flying speed and the flight altitude, may be accompanied by the phenomenon of hysteresis (also see [18] where it has been shown that the type of reflection of shocks in an underexpanded wake is dependent on the prehistory of flow).

Because of the great theoretical and practical interest in this problem, a manyfold study of all its aspects is carried out at present in analytical, computational, and experimental works whose detailed review is impossible here. Thus, the dynamics of reflection of an oblique shock wave from the axis of symmetry is related to the Guderley singularity in [19]. In [20], a four-wave structure is considered in modeling the diffraction of weak shocks ad it is inferred that the von Neumann paradox is only due to the insufficient resolving power of experimental measurements and numerical algorithms. In [21], in the problem on reflection of weak shock waves, it is stated on the basis of a certain analysis that a "very small portion of supersonic rarefaction flow" has been found behind the point of meeting of the incident and reflected Mach shocks. The fundamental nonstationary of the process of von Neumann reflection in incidence of a shock wave on a wedge to form a triple configuration is stated in [22]. Another group of works investigates the problem of nonuniqueness of "Mach and regular reflections" on the basis of a complex experimental and computer modeling, including that with the use of high-resolution schemes [23, 24] (of high order of accuracy); turbulence is allowed for in [25]. In investigating the diffraction of a strong shock wave on the surface with angles of inclination much smaller than critical ones, it has been found in [26] that the occurrence of Mach reflection is retarded and we have the regime of a "forerunner" of regular reflection (which is, generally, inconsistent with the von Neumann theory for a perfect gas). It is of considerable interest to study the nonuniqueness of numerical solutions with variation of different physical and algorithmical parameters of the problem [27–29].

Wave structures of two types (regular reflection and Mach reflection) are investigated, as a rule, under the assumption that the physical properties of a gas flow are constant in traversal of the entire system of shock waves, i.e., the model of an ideal polytropic gas with a constant value of the adiabatic exponent γ throughout the flow region is used. However actual processes (see [30, 31] whose study involves intensified development of hypersonic aircraft strongly call for the extension of this physical model. Since the problem diagrammatically shown in Fig. 2 models flow at entry into the air intake of a hypersonic ramjet engine, the level of knowledge of the regimes of this flow, the prediction of regular-to-Mach reflection transitions and, conversely, Mach-to-regular reflection transitions, and also the answer to the question of which one of the two types of shock-wave structures is realized in the domain of nonuniqueness of solution and what factors exert an influence on this, are extremely important in creating a system for control of the regime of burning of a fuel for stable operating of the entire propulsion system.

The nonuniqueness of shock-wave structures formed at entry into the air intake of a hypersonic ramjet engine has been investigated in [28–31] with allowance for the actual properties of the gas; the physical aspects of transition of on type of interaction into another have been investigated in [30]. We emphasize that computer modeling was carried out in [28–31] with allowance for the change in the properties of the gas medium on shock waves under the conditions of the Earth's atmosphere with a change in the adiabatic exponent of the gas from 1.2 to 1.4 as a function of the specific p and T values determined by the altitude of flight, the flying speed, and the geometric parameters of the diffuser (angles of downwash).

This work substantially extends the γ range under study. Here the modeling is carried out virtually throughout the range of a possible variation in $\gamma_{\rm eff}$ from 1 to 5/3.

Needless to say, this, on the one hand (in the context of physics), cannot be directly referred to the problem of motion of a body in a certain definite gas medium, since the adiabatic exponent can vary in such a wide range in none of the gases. However, on the other hand, such a formulation is very useful from the mathematical viewpoint, since it totally determines the degree of influence of $\gamma_{\rm eff}$ on the resulting solution through the permissible range of variation.

In particular, we should point to the significant difference in the high-velocity motion of an aircraft in the atmosphere of the planets: the diatomic oxygen-nitrogen atmosphere of the Earth, the triatomic atmosphere of Mars, the high-molecular-weight methane-hydrogen atmospheres of Jupiter and Saturn, etc. Therefore, a study of such a motion (even in a model formulation) seems very useful, since it provides exhaustive information on all possible changes in the shock-wave patterns in the gas flow.

Procedure of Investigation. The technique of shock polars is quite conveniently used for analysis of wave occurring in interaction of the incident shock waves i_1 and i_2 and determined by the formation of reflected shock waves r_1 and r_2 of different types (regular reflection and Mach reflection). We note that this technique is widely used in solving problems of classical aerodynamics (under the assumption that the properties of the gas medium are invariant) but some of its aspects have their distinctive features in the case of an abrupt change in these properties on a shock wave (see [28]). By using the shock-polar technique, the complicated mathematical method of simultaneous mathematical method of simultaneous solution of a few nonlinear algebraic equations (their number is determined by the number of interacting shock waves) relating the values of the parameters ahead of the front of each shock and behind it may be replaced by a clear graphical method of obtaining the solution, when it is necessary to select solutions (because of their nonuniqueness). This method makes the process itself of obtaining solutions and their analysis much more clear and logical, and selection of the necessary solution (in the case of its nonuniqueness) is much less troublesome.

By the polar of a shock wave, or simply the shock polar, we mean the relation relating the angle of deflection of the flow θ and the pressure ratio $\xi = p_+/p_-$, where p_+ and p_- are respectively the pressure behind the shock front and ahead of it, for the parametric dependence on the Mach number M_0 and the effective adiabatic exponent $\gamma_{\rm eff}$; this relation is written in the form $f(\theta, \xi, M_0, \gamma_{\rm eff}) = 0$. This plot in the plane $(x, y) = (\theta, \xi)$ represents a closed curve which is called a shock polar. It is bounded by the values $\theta_{\min} \le \theta \le \theta_{\max}$ and $\varepsilon_{\min} \le \xi \le \xi_{\max}$ and is mirror-

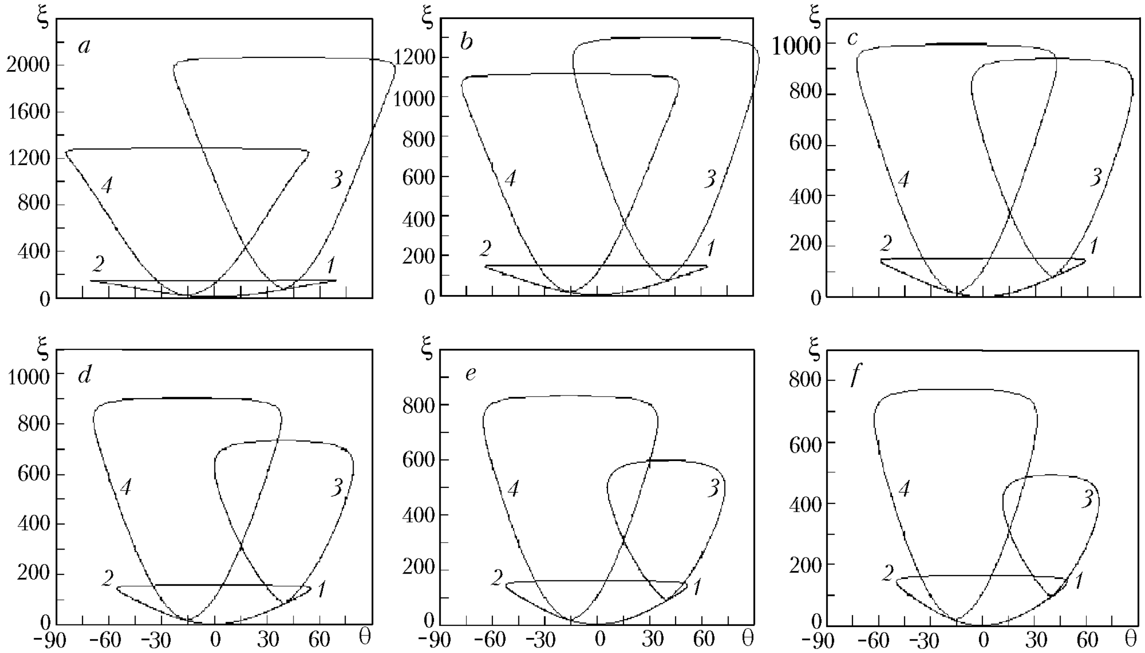

Fig. 3. Polars of two incident i_1 (1) and i_2 (2) and two reflected r_1 (3) and r_2 (4) shock waves. For high-molecular-weight gases: γ_{eff} = 1.05 (a), 1.10 (b), 1.15 (c), 1.20 (d), 1.25 (e), and 1.35 (f).

symmetric about the straight line $\theta_s = 0.5\ (\theta_{min} + \theta_{max})$. The specific form an a detailed analysis of shock polars with variation of the parameters have been given in [8, 28].

The shock-wave structures of the problem on question (Fig. 2) are determined by the parametric list $F = (\beta_1, \beta_2, M_0, \gamma_{eff})$. Different types of shock-wave structures may be formed with variation of the values of these parameters. Isolation of one parameter, e.g., β_2, which is selected as a "reference" one for investigation, is quite convenient for analysis. In the space of permissible β_2 values, there are two singular points $\beta_2^* = \beta_2^*(\beta_1, M_0, \gamma_{eff})$ and $\beta_2^{**} = \beta_2^{**}(\beta_1, M_0, \gamma_{eff})$ called respectively the lower and upper (since $\beta_2^* < \beta_2^{**}$) bifurcation points of the solution. These points determine the following ranges of shock-wave structures: only regular reflection is possible, if we have $\beta_2 < \beta_2^*$, both regular reflection and Mach reflection are possible, if $\beta_2^* \le \beta_2 \le \beta_2^{**}$, only Mach reflection is possible, if $\beta_2 > \beta_2^{**}$.

From the plots of the shock polars (see, e.g., Fig. 3), we may establish which of the shock-wave structures is realized for a certain set of the parameters even without knowing the numerical values of β_2^* and β_2^{**}. Let us write these conditions analogous to those given above in the same order but in a different formulation: of the polars r_1 and r_2 intersect inside the polar i_1, Mach reflection is impossible, if the polars r_1 and r_2 intersect outside the polar i_1, both regular reflection and Mach reflection are possible; if the polars r_1 and r_2 do not intersect, regular reflection is impossible.

We note that, in addition to determination of the boundaries of regimes, the shock-polar technique makes it possible to obtain the numeral characteristics of flows (see Fig. 2), such as the relative and absolute values of the pressure and the angles of deflection of the flow on the fronts of all the shocks, from which we may subsequently determine all the remaining gasdynamic parameters ad the angles of inclination of the shock waves throughout the flow region.

Discussion of the Results. Let us consider the influence of the effective adiabatic exponent γ_{eff} on the formation of one shock-wave pattern of flow or another (Fig. 2). An analysis will be carried out with successive variation of γ_{eff} from an almost minimum possible value of γ_{eff} to that maximum possible for the gas medium (order of the plots from 3a to 4f). We note that the minimum γ_{eff} values correspond to high-molecular-weight gases with a strong excitation of the vibrational degrees of freedom in their molecules. Thus, experiments with different working gases — freons 12 and 14, ethane, etc. — with low values of γ_{eff} have been carried out in [15, 16]. Significant differences of high-velocity (hypersonic) flows of such gases from the flow of "ordinary" air were found. The maximum γ_{eff} values

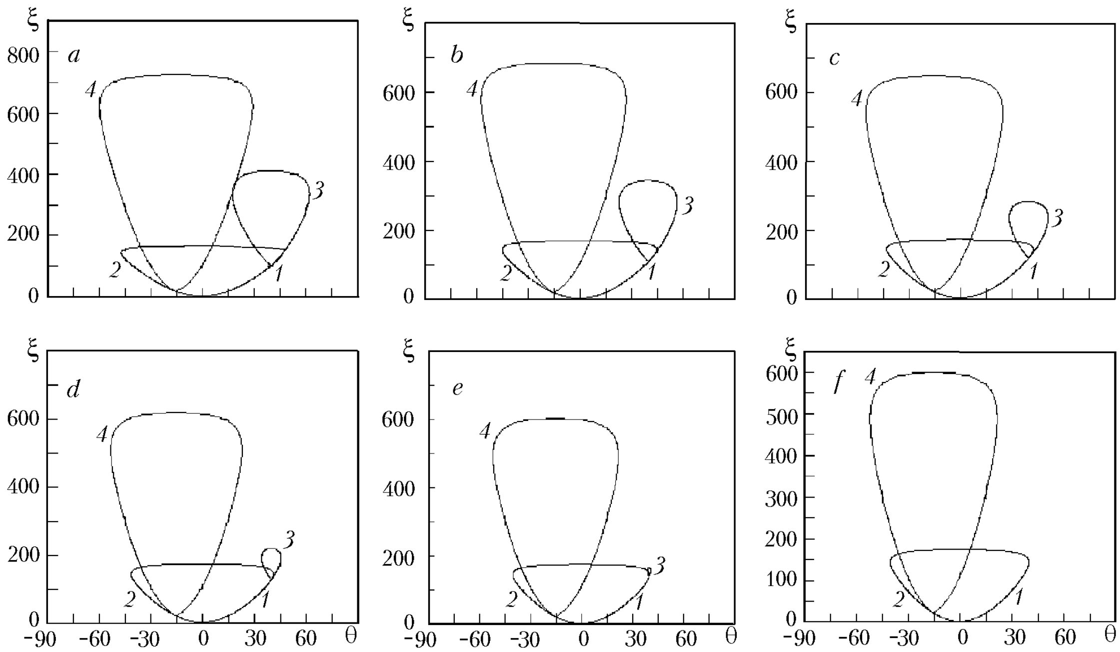

Fig. 4. Polars of two incident i_1 (1) and i_2 (2) and two reflected r_1 (3) and r_2 (4) shock waves. For low-molecular-weight gases: γ_{eff} = 1.35 (a), 1.40 (b), 1.45 (c), 1.50 (d), 1.53 (e), and 1.534 (f).

correspond, as has already been indicated above, to monatomic gases with unexcited or weakly excited electron shells and to a mixture of mon- and diatomic gases with unexcited vibrational degrees of freedom.

We should additionally dwell on the range of variation of γ_{eff}, which has been selected as that under study in the present work. The calculations were carried out for the ranges of variation in the effective adiabatic exponent γ_{eff} 1.05–1.534. Needless to say, this range is substantially wider than the range of applicability of a hypersonic ramjet engine but it is of great interest and has been selected for investigation for the following two basic reasons. First, it is, certainly, of prime importance to gain a global idea of the idea of investigations and to know what may occur beyond the actual subrange of operation of the hypersonic ramjet engine in the near and (to a smaller degree) relatively distant regions of the total range of variation of γ_{eff}. This knowledge is quite useful from both the practical (possible consequences of moving out of the nominal operating regime of a hypersonic ramjet engine) and purely theoretical (how wide the range of applicability of the model is) viewpoint. Furthermore, the results given below allow a certain degree of interpolation and, with certain caution, extrapolation by the parameters. Second, this analytical work has an applied continuation: a databank in which results of the mathematical modeling of this problem are placed is created on its basis. The system for control of the databank calls for the "rectangularity" of data arrays, even through part of them is of little interest. The use of modern computer technologies will ensure necessary results for any γ_{eff} values through spline interpolation by the reference values in the databank without carrying out calculations.

Initial data. Computer modeling of the shock-wave structures shown in Fig. 2 was carried out for the following values of the governing parameters: $\beta_1 = 40^o$, $\beta_2 = 15^o$, and $M_0 = 12$. Selection of precisely these values for representation of the results in the present work (the program complex ensures calculations with any physically meaningful values) was determined by the following considerations.

The freestream Mach number equal to 12 belongs to the nominal range of the experiments carried out. Thus, flight test of the hypersonic ramjet engine of an X-43 aircraft were carried out in late 2004 under the Hyper-X program; the aircraft reached an altitude of 33 km and was accelerated to a speed of 3 km/sec by the Pegasus rocket which in turn started onboard a B-52B airplane flying at an altitude of 12 km.

The diffuser angles equal to 40^o and 15^o were selected for the following "aeromechanical" reasons. An angle of 40^o is close to the limiting one for which a shock attached to the top of a wedge may exist and the shock-wave structures shown in Fig. 2 may be formed. A small angle of 15^o ensures (almost without exception) the regular type of reflection. Thus, all the flow structures of practical interest are formed by the angles lying in this range. Analyzing the shock polars in Figs. 3 and 4 enables us to approximately determine the flow structures for other values of β_1 and β_2, mentally moving the r_1 and r_2 polars to the reference points of the i_1 polar corresponding to these values. Needless to say, such a visual approximation will not ensure exact numbers but may provide a qualitative idea of the change in the shock-wave flow structure with variation of β_1 and β_2.

Furthermore, there is also another reason for the selection of these precisely values of the angles β_1 and β_2. Let a symmetric inlet of the diffuser of the hypersonic ramjet engine be designed. However, the symmetry of flow is broken in motion of the aircraft on the portions of ascent or descent of its trajectory at the freestream angle of attack. In particular, the problem presented may be considered as flow in a symmetric air intake with $\beta_1 = \beta_2 = 27.5^o$ at an angle of attack of 12.5^o.

Representation of the Results. As has already been noted above, the results will be analyzed with the use of the shock-polar technique (also, see [8, 28, 30, 31]). The polars of shock waves (Fig. 2) — two waves incident (i_1 and i_2) onto the diffuser from the angles on entry into it and two reflected waves (r_1 and r_2) with different types of interaction between them (Mach reflection and regular reflection) — are presented in Figs. 3 and 4 uniform in form. The polars i_1 and i_2 coincide for this set of governing parameters. We recall that the polar r_1 rests, with its zero point, on the polar i_1 at the point $\theta = \beta_1 = 40^o$, whereas the polar r_2 rests on the polar i_2 at the point $\theta = -\beta_2 = -15^o$.

Polar i_1 (and i_2). The dimensions of the polar are dependent on the value of γ_{eff}: the lower the value of γ_{eff}, the larger (all other governing parameters being equal) the width θ_{max} (more precisely, the halfwidth, but we will use this term for the sake of brevity) and the height ξ_{max} of the polar. For hypersonic flows $M_0 >> 1$, we may write the asymptotics

$$\theta_{max} = \arctan \frac{1}{\sqrt{\gamma_{eff}^2 - 1}}, \quad \xi_{max} = \frac{2\gamma_{eff} M_0^2}{\gamma_{eff} + 1}. \tag{7}$$

The polar height ξ_{max} is weakly dependent on γ_{eff}, since the coefficient $2\gamma_{eff}/(\gamma_{eff} + 1)$ in (7) changes in the range of variation in $\gamma_{eff} \in [1, 5/3]$ only slightly: from 1 to 1.25. The polar width θ_{max} is dependent on γ_{eff} very strongly and changes from 70^o for $\gamma_{eff} = 1.05$ (Fig. 3a) to 40^o for $\gamma_{eff} = 1.534$ (Fig. 4f). This fact is quite significant.

Relative Dimensions of r-Polars. It is of interest to compare all the 12 variants of the results of solution of the problem of this computational experiment; they are presented in Figs. 3 and 4. We may draw a few important conclusions from these plots. First, different degrees of influence of γ_{eff} on the solution are noteworthy. For low γ_{eff} values, the polar r_1 is larger than the polar r_2 in both width and height. We note that each individual r polar decreases with growth in γ_{eff} (this has already been discussed in the previous section), but the rate of decreases in the r_1 polar is much higher than that of the r_2 polar. Therefore, the polar r_1 becomes comparable to the polar r_2 already to a value of $\gamma_{eff} \approx 1.13$ and continues to rapidly decrease further. The r_1 polar disappears for a certain critical value of $\gamma_{eff} = \gamma_{eff}^* = 1.534$. We note that the width θ_{max} of the i_1 polar becomes equal to 40^o (and decreases further with increase in γ_{eff}). Therefore, the relation $\theta_{max} < \beta_1$ becomes true and the reference point of the r_1 polar goes beyond the i_1 polar. The disappearance of the polar r_1 means that the shock wave i_1 becomes, from the attached one, a wave departed from the vertex of the angle β_1. As a result, the existence of none of the shock-wave structures shown in Fig. 2 — neither the Mach wave nor the regular one — turns out to be possible. A different, more complex shock-wave pattern must occur (this question is beyond the scope of the present work).

The dimension of the r_2 polar do not change so strongly: ξ_{max} decreases approximately from 1300 for $\gamma_{eff} = 1.05$ to 600 for $\gamma_{eff} = 1.53$. The swing of the r_2 polar changes noticeably but not very substantially (compared to r_1): its halfwidth $(\theta_{max} - \theta_{min})/2$ decreases from 68 to 36^o.

Thus, numerical experiments in the cycle of variation of γ_{eff} have shown a high and very different degree of influence of this exponent on the flow structure for different values of β_1 and β_2.

Domain of Nonuniqueness of the Solution. Point of Intersection of the $(r_1 \times r_2)$ Polars. An analysis of the results of numerical modeling (Figs. 3 and 4) in the context of determination of the type of shock-wave structure occurring (for a certain γ_{eff}) leads us to the following conclusions. The domain of nonuniqueness of the solution (possibly, of both Mach reflection and regular reflection) for these values of β_1, β_2, and M_0 is in the range $\gamma_{eff} \in [1.05, 1.35]$. Here, the interval of γ_{eff} values used in the numerical experiment is given.

Additional calculations have shown that when $\gamma_{eff} \to 1$ we have dualism of the solution as before; only the shapes of all the polars and their dimensions change. The polars i_1 (and i_2) are flattened and break in the limit $\gamma_{eff} \to 1$ and are represented by a straight line (see [8] for details). The width and height of the polars r_1 and r_2 increase; the growth in the r_1 polar is much more rapid than that in the polar r_2. This could be predicted a priori, without calculations, from the pattern of change in the r_1 and r_2 polars in Fig. 3.

The type of tangency of these polars (see Fig. 4a) is of great interest. Generally speaking, these are always two points of intersection of the $(r_1 \times r_2)$ polars. The lower point corresponds to a weak solution, whereas the upper one corresponds to a strong solution (see [28] for details). It is generally believed that only a weak solution is realized in the case of dualism of the solution (see, e.g., the problem on incidence of a supersonic flow on a wedge or a cone). However, this question remains open theoretically as before.

The following important circumstance is noteworthy (see Fig. 4a). The polar r_1 is tangent to the polar r_2 at its upper part, i.e., at the subdomain of its "strong" shock-wave solutions. We recall that the polar written in the form $\xi = \xi(\theta)$ is a two-valued function, i.e., two solutions exist for each $\theta \cup [\theta_{min}, \theta_{max}]$: $\xi_1(\theta) = \xi_2(\theta)$, where $\xi_1 < \xi_2$, except for the points $\xi_1(\theta_{min}) = \xi_2(\theta_{min})$ and $\xi_1(\theta_{max}) = \xi_2(\theta_{max})$. The upper branch $\xi_2(\theta)$ is called the strong solution, whereas $\xi_1(\theta)$ is called the weak solution. We emphasize that, despite the similarity of terminology, we should differentiate between strong and weak "*isolated*" shock-wave solutions (upper and lower branches of *individual* polars) and strong and weak solutions of the *entire problem* (upper and lower points of intersection of *different* polars). This problem has been analyzed in detail in [28] where, in particular, the question of nonuniqueness with a theoretical possibility of existence of 14 different solutions was considered.

Thus, the strong solution of the regular interaction of shock waves is realized at the instant of "contact" for $\gamma_{eff} = 1.35$. Moreover, there is a very small interval of $\Delta\gamma_{eff}$ (from 1.348 to 1.352), in which *both* points of intersection simultaneously belong to the upper branch of the r_1 polar and the lower branch of the r_2 polar, i.e., we have two shock-wave solutions, strong ones for the r_1 polar and weak solutions for the r_2 polar. Next, at γ_{eff} decreases, the situation becomes standardized: of the two points of intersection of the $(r_1 \times r_2)$ polars, the upper point is formed by intersection of the upper branches, whereas the lower one is formed by intersection of the lower branches. However, the presence of an actually existing $\Delta\gamma_{eff}$ interval, even if small, makes the meaning of the above-mentioned popular opinion that "of the possible solutions, it is the weak one that is always realized" quite ambiguous.

Domain of Uniqueness of the Solution. Point of Intersection of the $(i \times r)$ Polars. Only one solution — a Mach shock-wave configuration (see Fig. 2b) — may exist due to the nonintersectability of the polars r_1 and r_2 in the range of variation of $\gamma_{eff} > 1.35$ in the case where the solution is existent at all.

Let us analyze the change in the shock-wave flow pattern with increase in γ_{eff} (Fig. 4). We consider two aspects of this question: decrease in the dimensions of the polar and change in the position of the points of intersection of the $(i_1 \times r_1)$ and $(i_2 \times r_2)$ polars, paying special attention to the decrease in their number. The swing of the polars is determined by the physics of the problem, i.e, by the value of γ_{eff}. The higher the γ_{eff} value, the smaller the width and height of the polars. The height of the i polars ξ_{max} remains virtually constant in this subrange of variation of $\gamma_{eff} \cup [1.35, 1.534]$, but the width θ_{max} decreases from 62 to 40^{o}. The quantity $\theta_{max}(\gamma_{eff})$ tending precisely to 40^{o} has a decisive effect, as has already been mentioned above, on the reformation of the flow pattern, since one angle of entry into the air intake is $\beta_1 = 40^{o}$. This circumstance leads to a rapid decrease in the dimensions of the r_1 polar, to the point of its total disappearance: degeneration to a point for $\theta_{max}(\gamma_{eff}) = \beta_1$. Once this criterion has been attained, the existence of shock-wave flow patterns — either Mach reflection (Fig. 2b) or, especially, regular reflection (Fig. 2a) — becomes impossible.

We consider the second aspect: change in the position of the points of intersection of the $(i \times r)$ polars, which is substantially dependent on the values of the angles β_1 and β_2 determining the position of the reference points of the r_1 and r_2 polars on the polars i_1 and i_2 respectively. In most of the range of variation of γ_{eff} under study and the values of other parameters, selected for numerical experiments there are only two points of intersection of the $(i \times r)$ polars in each. The left $(i_1 \times r_1)$ and right $(i_2 \times r_2)$ points are called weak solutions, whereas the other two are called strong solutions. The terminology has been derived from the fact that the absolute values of the abscissas (θ_1^{w} and θ_2^{w}) for "weak" points are lower than those for "strong" points (θ_1^{st} and θ_2^{st}) for each pair of points of intersection of the polars $(i_1 \times r_1)$ and each pair of $(i_2 \times r_2)$ points. These abscissas are the angles of direction of the flow θ_3 and θ_4 (see Fig. 2) at exit from a system of shocks.

There is an opinion (relying on aerophysical experiments) that if two solutions are theoretically possible, it is the weak solution that is realized: $\theta_3 = \theta_1^w$ and $\theta_4 = \theta_2^w$. In other words, the angle of convergence of the flow at exit from the system of shocks $\Delta\theta = \left|\theta_3 - \theta_4\right|$ must be the minimum of the two possible angles. This angle plays a very important role in the organization of the flow downstream in the air intake, in the engine's duct. The system of shock at entry (Fig. 2) compresses the flow, and the degree of this compression at exit flow it is characterized by an angle $\Delta\theta$ in addition to p and T. Next, additional shocks turning flow along the duct channel, i.e., compensating for the influence of $\Delta\theta$ may be formed in the flow.

We consider the behavior of the function $\Delta\theta(\gamma_{eff})$ in the interval $\gamma_{eff} \cup [1.35, 1.53]$. The components $\Delta\theta$ of the functions $\theta_3(\gamma_{eff})$ and $\theta_4(\gamma_{eff})$ change in significantly different manners. The second of them is nearly constant: $\theta_4(\gamma_{eff}) \approx 5^o$ (changes from 4 to 6^o). The first changes significantly: from 30^o to 40^o. On quite a long portion $\gamma_{eff} \cup [1.05, 1.4]$, we have $\theta_3(\gamma_{eff}) \approx 30^o$, i.e., the function is virtually constant. This corresponds to behavior of $\Delta\theta(\gamma_{eff})$, too: this angle is nearly constant on the first interval given above ($\Delta\theta(\gamma_{eff}) \approx 25^o$) and sharply increases to 35^o on the second, much shorter interval $\gamma_{eff} \cup [1.4, 1.53]$. We note that, on the second interval, the question of selection between the strong and weak intervals for determination of θ_3 ceases to be pressing, since there is only one point of intersection of the $(i_1 \times r_1)$ polars for $\gamma_{eff} > 1.35$.

Domain of Absence of Solutions. When $\gamma_{eff} > 1.534$ the width of the polar i_1 polar becomes smaller than the angle of the upper wedge: $\theta_{max} < \beta_1$. The meaning of this criterion is that no single shock transition is capable of turning the flow by an angle larger than β_1. What this means is that there can be no shock with a rectilinear front attached tot he top of the wedge. A shock wave with a curvilinear front, departed from the wedge's top, is formed.

Thus, there can be no shock-wave patterns whose diagrams are presented in Fig. 2 with rectilinear fronts of all shocks (except for the central shock in Mach interaction with the curvilinear front). What this means from the viewpoint of physicochemical processes is that there are gas media in which such shock-wave regimes are possible and impossible. They are allowed by high-molecular-weight gases and not allowed by monatomic or low-molecular-weight gases. This should be taken into account in creating systems for control of hypersonic ramjet engines designed for operation on gas media with a wide range of variation in their properties.

Conclusions. We have investigated shock-wave structures occurring at entry into the duct of the engine of a hypersonic aircraft and the influence of the gas-medium parameters determining flow behind the fronts of oblique shocks incident into the diffuser with different types of interaction (Mach or regular) on the process of formation of these structures. To allow for the actual properties of the atmosphere we used the procedure of the effective adiabatic exponent; this procedure makes it possible to determine the topology of shock-wave patterns and to calculate the gas- and thermodynamic parameters in different zones of flow between the shock fronts in a wide range of governing parameters.

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NOTATION

c_p, specific heat at constant pressure; c_v, specific heat at constant volume; f, number of excited degrees of freedom of one gas component; F, level of excitation of the gas medium; g, statistical weight; k, Boltzmann constant; M, Mach number; p, pressure, atm; R, gas constant; T, temperature, K; V, specific volume; W, potential of the physical process; α, specific concentration of the component of the gas medium; β, angle of the wedge, deg; γ, adiabatic exponent; δ, angle of the wake; φ, angle of inclination of the shock; ξ, ratio of the pressures behind the shock and ahead of it; θ, angle of deflection of the flow; i, incident shock, m, central shock; r, reflected shock; S, wake. Subscripts and superscripts: 0, 1, 2, 3, and 4, Nos. of flow zones; + (plus) and − (minus), parameters behind the shock front and ahead of it; eff, effective value; i, component of the gas medium; max and min, maximum and minimum values; s, mean value; st and w, strong and weak solutions.

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